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PRELIMINARY STUDY OF ICE FORMATION IN A VERTICAL PIPE

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "PRELIMINARY STUDY OF ICE FORMATION IN A VERTICAL PIPE" submitted by CHRISTOPHER GORT in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

A theoretical and experimental investigation is presented for the problem of freezing inside a long vertical cylinder without flow and an experimental investigation is presented for the problem with flow. The solution for the theoretical problem is an approximate solution for arbitrary departures of the cylinder wall temperature from the freezing temperature. The experimental results of the problem without flow agree well with the theoretical results. From the experiments with flow it is found that for increasing flow rates the equilibrium ice thickness decreases.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

FIGURE

NOMENCLATURE

A, a	position of interface with respect to center line
c	constant
C_p	specific heat
κ	thermal diffusivity
K	thermal conductivity
L_f	latent heat of freezing
L	length of test section
n	index
R, r	radial distance from center line of cylinder
Re	Reynolds number
ρ	density
Ste	Stefan number
T, θ, ϕ	temperature
t, τ	time
Z, z	distance from leading edge along center line
θ_1	intercept of τ_{lim} vs n plot (θ_w at t = 1)
$\bar{\theta}_c$	time averaged temperature difference between wall temperature and 32°F

SUPERSCRIPT

$\cdot$	differentiation with respect to time

SUBSCRIPTS

o	at 32°F
w	wall
c	characteristic
I	ice
lim	time for $a = 0$
∞	infinite time

CHAPTER I

INTRODUCTION

The phenomenon of solidification can be observed and utilized in many different processes such as freezing of foods, casting of metals and desalination of water. It is also an important factor in construction and development in the Canadian North and its effects are felt more sharply there than in more Southern regions. Ice formation in water mains can disrupt normal living severely. Ice formation in the cooling system of cars can play havoc with the motors of these vehicles. In the permafrost regions of the North difficulties are encountered in the building of airfields, houses and office buildings. The latter have to be insulated from the soil to prevent heat leakage which may turn the area below the house or office building into a mudpool. In other areas wet soil must be stabilized by freezing. Cylindrical piles are located in the soil and the pile is maintained at a temperature below the freezing point in order to freeze the surrounding soil. Another area where solidification can be a problem is space technology. Space vehicles and satellites are subjected to extreme environmental temperatures and solidification may occur in hydraulic lines if proper precautions are not taken.

Considering the importance of solidification of a liquid flowing through a pipe it is surprising to find that so little research appears to have been carried out in this field. Brush [1] in 1916 discussed the principles governing freezing of water in mains and concluded, as would be expected, that the formation of ice in water mains is dependent upon the

temperature of the water and the flow rate. He stated that when the wall temperature was reduced below the freezing temperature a coating of ice was formed on the inside surface of a water main. Brush felt that for reduced flow rates the water was more easily cooled, ice thickness would increase, and therefore complete freeze-up of water mains located above the frostline could be prevented only by maintaining such a flow rate that the cooling rate would be insufficient to allow the water temperature to fall below the freezing temperature. Brush did not present a theoretical analysis or experimental data in his paper.

In 1939 Pekeris and Slichter [2] attempted a solution to the problem of ice formation on a long cylinder whose surface temperature was time dependent. They were able to solve the steady state problem and determine the first order correction to the steady state solution which was found to be small. Their solution for the conduction equation was based on the fact that for the ice-water system in consideration, the ratio of the sensible heat to the latent heat was small. It enabled them to use the steady state solution as an approximation.

London and Seban [3] in 1943 described a general approximate analogue method for analyzing the problem of freezing in liquids bounded by surfaces of various geometries. They presented algebraic and graphic solutions for the rate of ice formation for common shapes, i.e. cylinders, spheres, and plane surfaces, and estimated the degree of approximation of these solutions. The results were compared with specific applications in ice manufacture and quick freezing of food. It was found that the total freezing times as determined by the theoretical analysis

were comparable to the freezing times obtained experimentally. Their theoretical solutions consisted of the construction of a thermal circuit, considering the sensible heat negligible relative to the latent heat of freezing. London and Seban concluded that for the slab problem the approximate solution adequately approached the behavior of the actual system.

Allen and Severn [4] in 1962, using the method of relaxation, treated the problem of the cylinder. They found that the total time needed for solidification was half the amount of time needed for what they considered the upper limit and twice the amount of time needed for the lower limit: the upper limit being a slab of thickness R and the lower limit a slab with the same area of surface as the cylinder and the same volume, thus having a thickness of R/2. Because of this they concluded that their results were within 4% of the true values.

In the same year Poots [5] applied integral methods to the solution of problems involving the solidification of liquids initially at the fusion temperature. The integral methods were similar to those used in the solution of the boundary layer equations in fluid dynamics. Series solutions of the thermal equation giving the location of the solid-liquid interface, for small time, were derived for the circular cylinder and were used to estimate the accuracy of the approximate integral methods. These results were compared with Allen and Severn's work and were seen to be satisfactory. The time for total solidification using Allen and Severn's approach was $\tau' = 0.47$* as compared to $\tau' = 0.40$ for the Karman-Pohlhausen method and $\tau' = 0.52$ for the Tani method used in Poots' work, all for a value of Ste = 0.64. Neither Allen and Severn nor Poots

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* $\tau' = \dfrac{\tau}{Ste}$.

compared their results with experimental data.

More recently, Zerkle and Sunderland [6] presented a paper at the ASME-AIChE Heat Transfer Conference and Exhibit in Seattle dealing with the effect of solidification upon steady laminar flow heat transfer in a tube. They assumed steady state conditions and a uniform wall temperature. It was found that ice growth depended on the flow rate and the amount of superheat present in the water. For a given flow rate and a fixed amount of superheat the interface comes to equilibrium when the heat transfer through the ice equals the heat transfer through the water. The experiments were carried out in horizontal tubes and it was estimated that free convection affects the solid phase shell to a considerable extent. The influence of free convection was found to decrease for an increase in distance from the leading edge.

The purpose of this thesis is to study the problem of freezing inside a long vertical cylinder with and without a forced internal flow. For no-flow conditions, an approximate theoretical solution for interface position versus time is presented for any variation in the wall temperature. Graphs are plotted to make the solutions for different power-law variations of the wall temperature readily available.

Experimental work is conducted for no flow and flow in the test section and in particular the influence of flow and superheat on the ice formation is studied. In the Appendix an analysis of higher approximations for no-flow conditions is developed.

CHAPTER II

THEORETICAL ANALYSIS

2.1 GOVERNING EQUATIONS

Consider quiescent liquid and its corresponding solid occupying the cylindrical domain shown in fig. 2.1. If the ice properties are assumed to be constant, the differential equation governing heat conduction within the ice region (for radial symmetry and no sources) is given by:

$$\frac{\partial^2 T}{\partial R^2} + \frac{1}{R}\frac{\partial T}{\partial R} + \frac{\partial^2 T}{\partial Z^2} = \frac{1}{\kappa}\frac{\partial T}{\partial t} \qquad 2.1\text{-}1$$

where T(Z, R, t) is subject to the conditions:

$$T(Z, R, 0) = T_i$$

$$T(Z, A, t) = T_o$$

$$T(Z, R_c, t) = T_w(t)$$

Taking a heat balance at the interface we find that the rate at which latent heat is withdrawn is equal to $-L_f \rho_I 2\pi A \frac{dA}{dt}$. Assuming the temperature of the liquid to be constant, there will be no flow of heat to or from the water and therefore the latent heat liberated must be conducted through the ice. The corresponding heat transfer rate from the interface is equal to

$$-K_I 2\pi A\left(\frac{\partial T}{\partial R}\right)_A$$

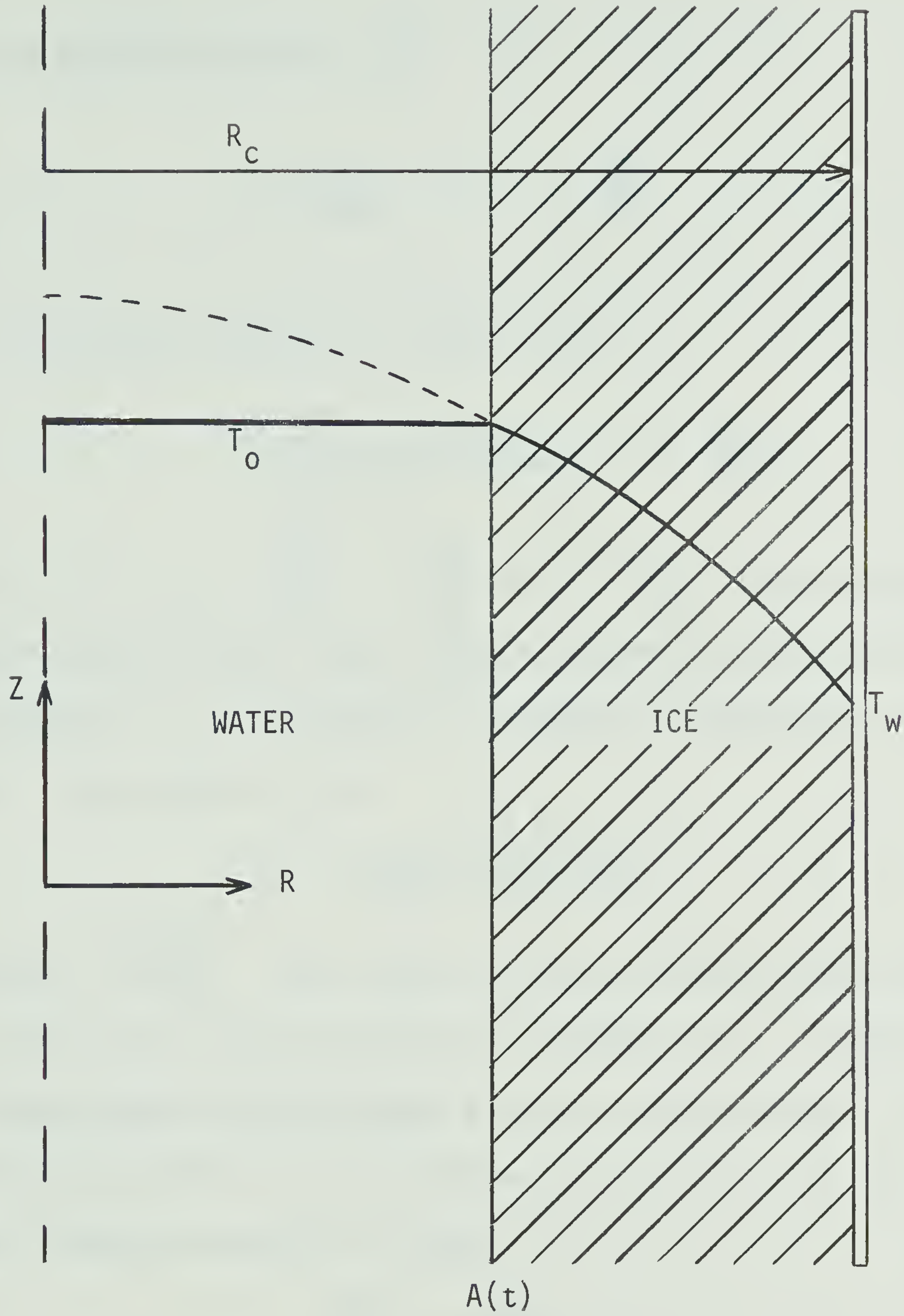

FIG. 2.1 COORDINATE SYSTEM

Thus the heat balance will be:

$$K_I \left(\frac{\partial T}{\partial R}\right)_A = + \rho_I L_f \frac{dA}{dt} . \qquad 2.1\text{-}2$$

Normalizing equation 2.1-2 will give

$$\left[\frac{t_c K_I \theta_c}{\rho_I L_f R_c^2}\right] \left(\frac{\partial \phi}{\partial r}\right)_{a(\tau)} = + \frac{da(\tau)}{d\tau}$$

where $\theta = T - T_F$, $\phi = \frac{\theta}{\theta_c}$, $a = \frac{A}{R_c}$ and $\tau = \frac{t}{t_c}$. The coefficient in square brackets cannot be very small without suppressing the basic feature of the problem i.e. ice formation. By taking it identically equal to unity* we may define t_c by:

$$t_c = \frac{1}{Ste} \left[\frac{\rho_I R_c^2 C_{pI}}{K_I}\right]$$

where $Ste = \frac{C_{pI} \theta_c}{L_f}$. Ste, which will be called the Stefan number, is the ratio of the sensible heat to the latent heat. For small values of Ste the latent heat dominates over the sensible heat so that in the cylindrical problem the ice interface advances very slowly. The normalized interface equation is then

$$\frac{da}{d\tau} = + \left(\frac{\partial \phi}{\partial r}\right)_{a(\tau)} \qquad 2.1\text{-}3$$

Normalizing equation 2.1-1 we obtain

$$\left[\frac{\theta_c}{R_c^2}\right] \frac{\partial^2 \phi}{\partial r^2} + \left[\frac{\theta_c}{R_c^2}\right] \frac{1}{r} \frac{\partial \phi}{\partial r} + \left[\frac{\theta_c}{L_c^2}\right] \frac{\partial^2 \phi}{\partial z^2} = \left[\frac{\theta_c}{\kappa t_c}\right] \frac{\partial \phi}{\partial \tau}$$

Multiplying by $\left[\frac{R_c^2}{\theta_c}\right]$ results in

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* Justified a posteriori (see fig. 2.3).

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + [\frac{R_c^{\,2}}{L_c^{\,2}}]\frac{\partial^2\phi}{\partial z^2} = [\frac{R_c^{\,2}}{\kappa t_c}]\frac{\partial\phi}{\partial\tau}$$

Assuming $R_c << L_c$, i.e. the pipe is very long, the term representing conduction in the z-direction becomes sufficiently small in comparison with the other terms that it may be neglected. Substituting for t_c the normalized conduction equation then becomes

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} = \text{Ste}\,\frac{\partial\phi}{\partial\tau} \qquad 2.1\text{-}4$$

The normalized initial and boundary conditions are then:

$$\begin{array}{lll} \tau = 0 & \phi(r,0) = 0 & \\ r = a(\tau) & \phi(a,\tau) = 0 & 2.1\text{-}5 \\ r = 1 & \phi(1,\tau) = -F(\tau) & \end{array}$$

2.2 SOLUTION FOR TEMPERATURE

In the problem under consideration the discussion will be restricted to Ste << 1. For this condition it is reasonable to use a perturbation expansion in order to arrive at a solution. The solution for equation 2.1-4 may then be taken as

$$\phi(r,\tau) = \phi_o + (\text{Ste})\,\phi_1 + (\text{Ste})^2\,\phi_2 + \ldots + (\text{Ste})^n\,\phi_n + \ldots \qquad 2.2\text{-}1$$

Substituting in equation 2.1-4 and grouping by powers of Ste we obtain an infinite set of equations

$$(Ste)^0 \quad : \qquad \frac{\partial^2 \phi_o}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_o}{\partial r} = 0 \qquad 2.2\text{-}2$$

$$(Ste)^1 \quad : \qquad \frac{\partial^2 \phi_1}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_1}{\partial r} = \frac{\partial \phi_o}{\partial \tau} \qquad 2.2\text{-}3$$

$$(Ste)^n \quad : \qquad \frac{\partial^2 \phi_n}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_n}{\partial r} = \frac{\partial \phi_{n-1}}{\partial \tau}$$

The solution for 2.2-2 is well known and given by

$$\phi_o = c_1(\tau) \ln r + c_2(\tau) \qquad 2.2\text{-}4$$

Using the boundary conditions 2.1-5 and substituting in 2.2-4 reveals the familiar form:

$$\phi_o = -F(\tau) \left[1 - \frac{\ln r}{\ln a}\right] \qquad 2.2\text{-}5$$

Solving equation 2.2-3 with the use of 2.2-4 and substituting ϕ_o and ϕ_1 in equation 2.2-1 we obtain:

$$\phi = C_1 \ln r + C_2 + \frac{Ste}{4} [(\dot{C}_2 - \dot{C}_1)\, r^2 + \dot{C}_1\, r^2 \ln r] + \ldots \qquad 2.2\text{-}6$$

Due to the difficulty in including higher order terms (see Appendix) only the zeroth approximation of the temperature will be used in the subsequent solution of the interface equation.

2.3 SOLUTION FOR INTERFACE DEPTH

Using the zeroth approximation of the temperature solution in the interface equation 2.1-3 we obtain

$$\frac{da}{d\tau} = + \left(\frac{\partial\phi_o}{\partial r}\right)_{a(\tau)} \tag{2.3-1}$$

and using equation 2.2-5

$$\left.\frac{\partial\phi_o}{\partial r}\right|_{a(\tau)} = \frac{F(\tau)}{a(\tau)\ \ln a(\tau)}$$

therefore $$\frac{da}{d\tau} = \frac{F}{a \ln a}$$

rearranging gives $$a \ln a\, da = Fd\tau$$

where a and F are functions of τ. Integrating the equation becomes

$$\frac{a^2}{2} \ln a - \frac{a^2}{4} = \int Fd\tau + \text{constant}$$

Using the initial condition

$$a(0) = 1$$

the constant is:

$$\text{constant} = - \int^0 Fd\tau - \frac{1}{4}$$

Hence $$\frac{a^2}{2} \ln a - \frac{a^2}{4} + \frac{1}{4} = - \int^0 Fd\tau + \int Fd\tau$$

i.e. $$\frac{a^2}{2} \ln a - \frac{a^2}{4} + \frac{1}{4} = \int_0^\tau F(\tau')\, d\tau' \qquad 2.3\text{-}2$$

or $$H(a) = G(\tau), \text{ say.}$$

The consequences of this are that the equation 2.3-2 can be solved for any variation of the wall temperature. A plot can be made of H(a) vs a and if the particular surface temperature variation is known $G(\tau)$ can be plotted versus τ. Comparing these two plots will give the position of the interface at any given time.

Analytically from the solution for $F_n(\tau) = \tau^n$ we may construct an arbitrary function

$$F(\tau) = \sum_{n=0}^{\infty} b_n \tau^n$$

so that $$G(\tau) = \sum_{n=0}^{\infty} \frac{b_n}{n+1} \tau^{n+1}$$

and therefore since $a \to 0$ as $\tau \to \tau_{lim}$ we have

$$\frac{1}{4} = \sum_{n=0}^{\infty} \frac{b_n}{n+1} \tau_{lim}^{n+1}$$

which may be inverted.

In particular when $F(\tau) = \tau^n$ equation 2.3-2 for a single value of n will read

$$\frac{a^2}{2} \ln a - \frac{a^2}{4} + \frac{1}{4} = \frac{\tau^{n+1}}{n+1} \qquad 2.3\text{-}3$$

and hence for this particular case

$$\tau_{lim} = \left(\frac{n+1}{4}\right)^{\frac{1}{n+1}} \qquad 2.3\text{-}4$$

A plot of τ_{lim} vs n is shown in fig. 2.2 which reveals that $0.25 \leq \tau_{lim} \leq 1.1$. A plot of a vs τ for different values of n is shown in fig. 2.3.

When $\theta_w = \theta_1 t^n$ the choice of θ_c is not obvious and t_c must be defined as follows. Let t_{lim} = actual time for complete freezing. Then the average temperature difference is given by:

$$\bar{\theta}_w = \frac{\theta_1}{t_{lim}} \int_o^{t_{lim}} t^n \, dt = \frac{\theta_1 \, t_{lim}^{\,n}}{(n + 1)}$$

or since τ_{lim} is the value of τ at a = 0

$$\bar{\theta}_w = \frac{\theta_1 (\tau_{lim} \, t_c)^n}{n + 1} = \theta_c \qquad 2.3\text{-}5$$

This should provide a reasonable definition for θ_c even if τ_{lim} is not known precisely. From the definition of

$$t_c = \frac{R_c^{\,2} L}{\kappa C_p \, \theta_c}^{*} \text{, substituting } \theta_c \text{ we obtain}$$

$$t_c = \frac{R_c^{\,2} L \, (n + 1)}{\kappa C_p \, \theta_1 (\tau_{lim} \, t_c)^n}$$

i.e.

$$t_c = \left[\frac{R_c^{\,2} \, \rho L (n + 1)}{K_I \theta_1 \, \tau_{lim}^{\,n}}\right]^{\frac{1}{n+1}} \qquad 2.3\text{-}6$$

From the above it is necessary to define a modified Stefan number Ste* by

$$Ste^* = \frac{C_p \, \theta_c}{L} = \frac{C_p \, \theta_1 (\tau_{lim})^n}{L(n + 1)} \left[\frac{R_c^{\,2} \rho L(n + 1)}{K_I \theta_1 (\tau_{lim})^n}\right]^{\frac{n}{n+1}} \qquad 2.3\text{-}7$$

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* where the subscript I has been omitted.

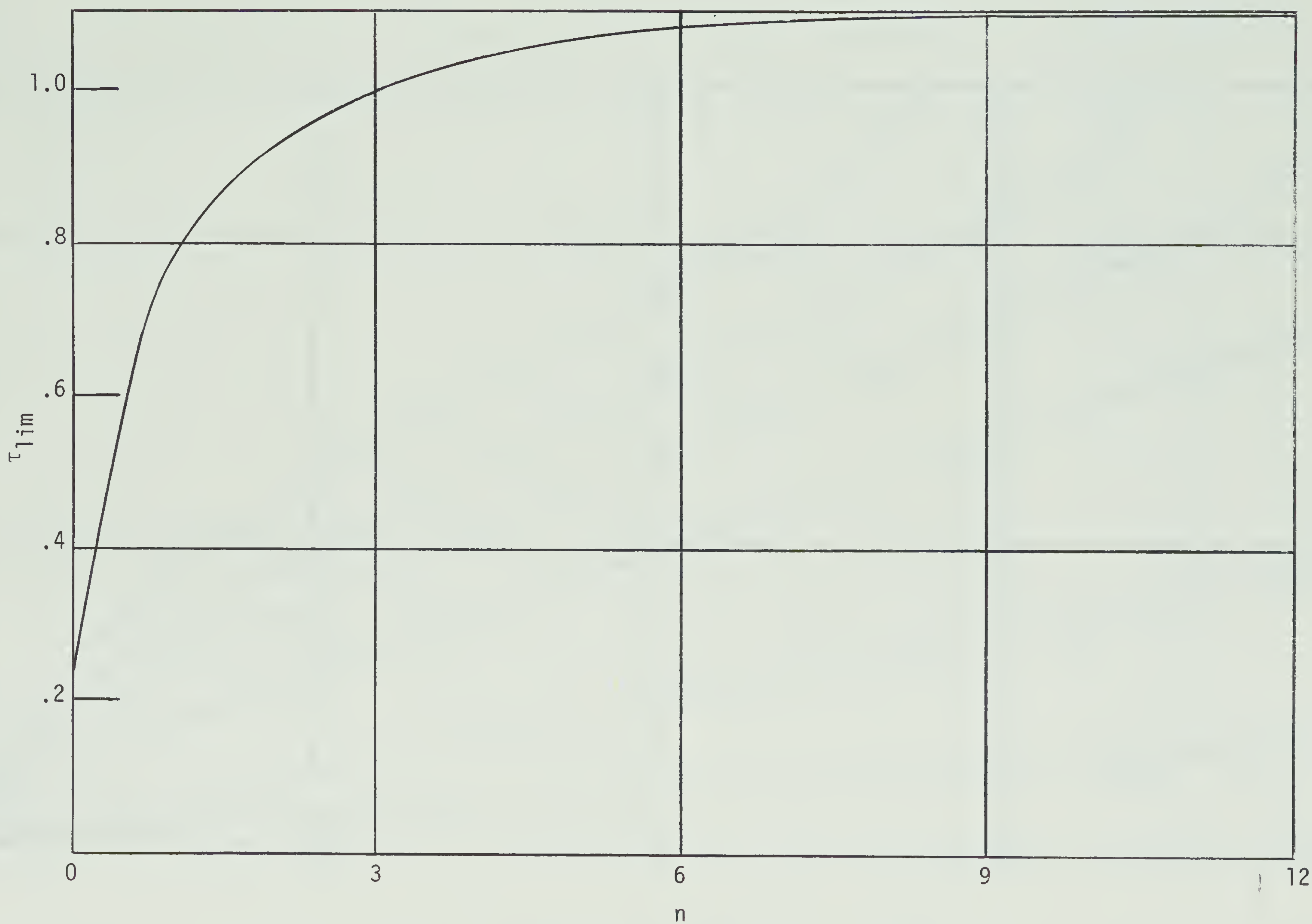

FIG. 2.2 VARIATION OF τ_{lim} WITH n

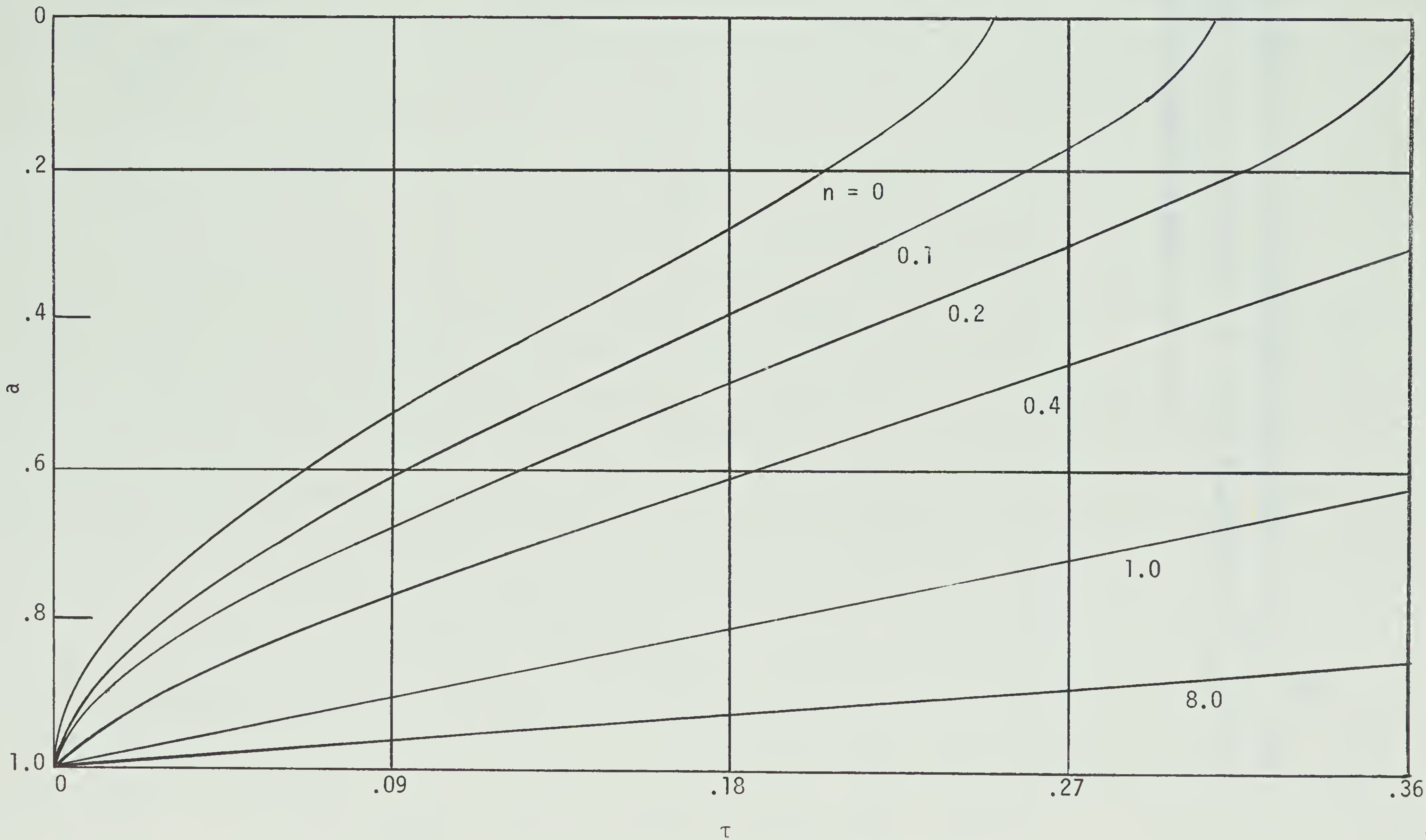

FIG. 2.3 COMPARISON OF INTERFACE PROFILES

Here again in view of the fact that τ_{lim} appears as $\tau_{lim}^{\frac{n}{n+1}}$ and $0 \leq \frac{n}{n+1} \leq 1$ it is evident that a precise knowledge of τ_{lim} will not materially affect the magnitude of Ste*.

CHAPTER III

EXPERIMENTAL WORK

3.1 DESIGN OF APPARATUS

The apparatus used for the experiments can be considered in two parts: a) the reservoirs and piping used for the storage and transportation of the coolant agent, antifreeze, and b) the reservoir and piping used for the storage and transport of the water (see fig. 3.1).

First the refrigeration system will be discussed. The coolant used was a mixture of 50% water and 50% antifreeze. It was stored in a closed reservoir 1-1/2 feet in diameter and 2 feet high. The reservoir contained a cooling coil connected to the first of two refrigerators. This refrigerator had a capacity of one ton per hour. The antifreeze-water mixture in the reservoir was kept at -20°F. The reservoir was connected to a 6 inch diameter, 3 foot high jacket by means of 1 inch diameter copper tubing, a three-way valve, and a pump. The jacket was also used as a reservoir in which, before each test, antifreeze and water were kept mixed at room temperature. About 1 foot downstream from the three-way valve a resistance bulb sensor (L 7033 A Balco) was located. The resistance thermometer was connected to a control mechanism (R 7087 D Honeywell resistance thermometer controller) which controlled the motor (M 904 Modutrol Motor) connected to the three-way valve (Honeywell Series 1616 Three-way Valve). The three-way valve in turn regulated the mixing of the -20°F coolant and the coolant from the jacket. The pump in the system (CAL Pump 2-875 L) pumped some of the coolant from the jacket into the main reservoir and some of it to

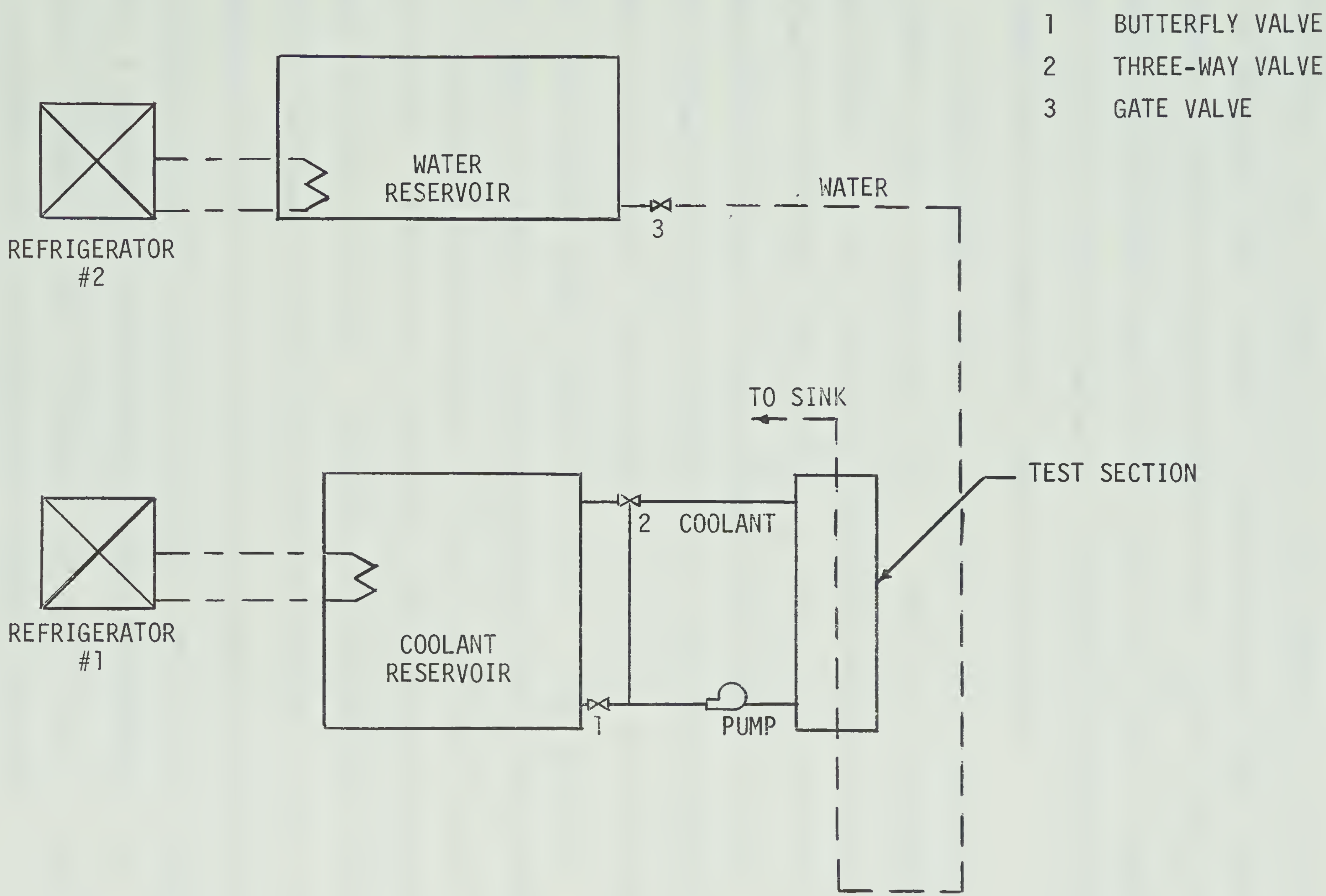

FIG. 3.1 SCHEMATIC DIAGRAM OF EQUIPMENT

the three-way valve, the amount going to each depending on the position of the three-way valve. In order to prevent any heat leakage into the system the reservoir was insulated with a 4 inch layer of urethane insulation and the copper pipes by a 3-1/2 inch thick layer of foamglass pipe insulation. These thicknesses were recommended by the suppliers as sufficient for the temperature ranges of the coolant.

The second system, used for the transport of water, consisted of a 2 foot x 4 foot x 2 foot header tank, a section of 1 inch diameter copper tubing, the entry length, the test section and the drainage pipe. The water in the header tank was cooled to 32°F by a series of coils around which an ice-bank of fixed thickness was located. The coils were attached to the second refrigerator unit which had a capacity of one half ton per hour. From the header tank the water was led through the 1 inch diameter copper tubing to a level of 11 feet below the water level in the tank. It was then channeled into a 6 foot section of 3 inch diameter copper tubing of which the last 3 feet served as the test section: the latter was surrounded by the previously mentioned jacket containing the coolant. The water left from the test section through a thick-walled rubber hose en route to the sink. At the discharge end, the flow rate was measured using a simple weighing technique. The header tank and all piping was insulated: the former was covered with a 2 inch layer of urethane and the latter surrounded by 1-1/2 inches of foamglass. These thicknesses were specified by the supplier as being sufficient for the temperature ranges of the water.

3.2 INSTRUMENTATION

The instrumentation presented some difficult problems since it had been decided to measure ice thickness inside a long pipe which was completely surrounded by a jacket containing coolant which, in turn, was well insulated. This effectively eliminated an approach through the walls of the pipe and left only the upper end of the test section as an area of approach. The solution was the use of a long 1/2 inch O.D. stainless steel probe with a rod inside. The probe was aligned with the centerline of the test section and could be moved from the lower to the upper extreme of the test section. At the bottom of the probe two slits were made in the wall. This allowed two arms, the lower ends of which were attached to the probe and the upper ends to the rod (see fig. 3.2), to move outwards by a controlled amount. The lateral movement of the arms was transferred to a vertical movement of the rod. At the upper end of the rod a pointer was fixed alongside a scale. In order to prevent the sharp edges of the joints of the arms from making impressions in the ice, round disks of plexiglass were located at the joints. A calibration curve of lateral movement divided by total diameter of copper pipe vs scale position was prepared using a micrometer. The calibration curve is shown in fig. 3.3. In addition to the above mentioned scale the stainless steel probe was marked at intervals of 2 inches in order to determine its position relative to the leading edge of the test section.

Further instrumentation consisted of two thermocouples located on the walls of the copper pipe in order to measure the wall temperature

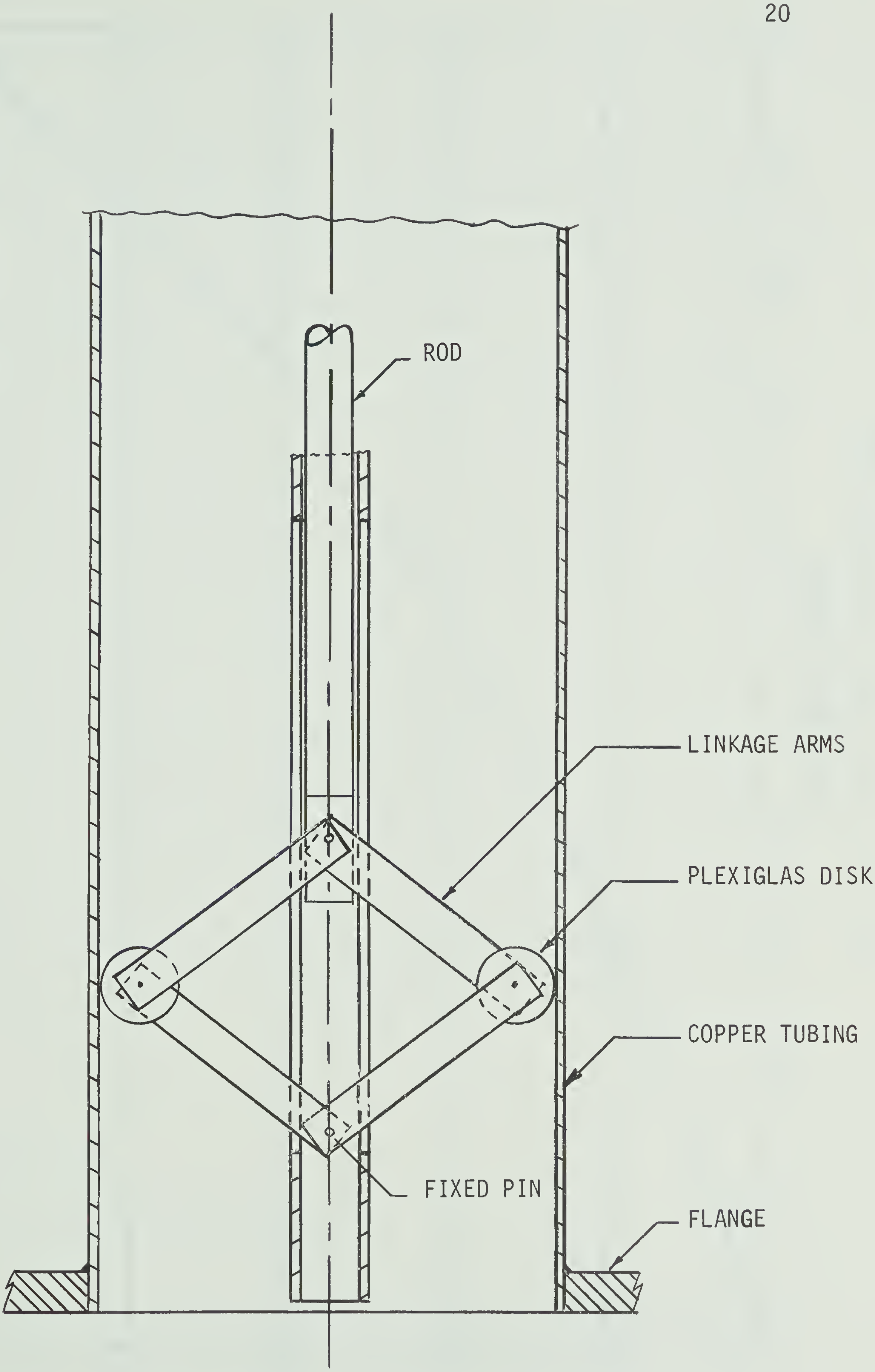

FIG. 3.2 PROFILOMETER IN FULLY EXTENDED POSITION

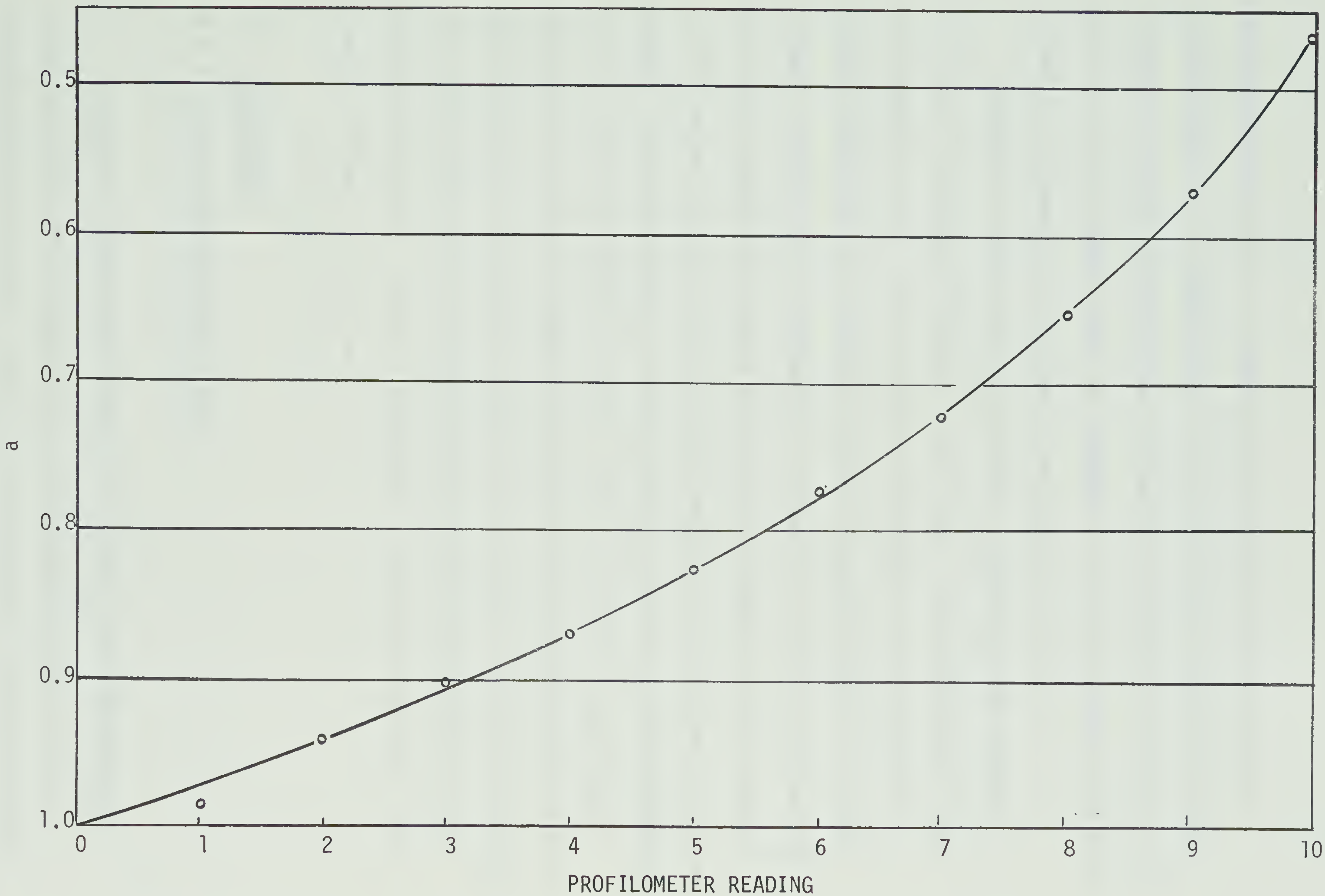

FIG. 3.3 PROFILOMETER CALIBRATION CURVE

during the test period. Readings were obtained with the use of a potentiometer. The temperature in the antifreeze reservoir was maintained at -20°F by a temperature thermostat connected to refrigeration unit #1. In the header tank an ice-bank of a given thickness was maintained by a thermostat attached to refrigeration unit #2.

In order to obtain the required wall temperature for the test section a resistance thermometer was located 1 foot downstream from the three-way valve. The resistance thermometer was connected to a control mechanism which compared the thermometer's signal to a preset value. If there was a discrepancy it energized the motor controlling the valve movement and opened the valve for either the -20°F coolant or the warmer coolant from the jacket depending on the polarity of the signal. The disadvantage of this system was that once the temperature in the jacket had reached a certain value, say 20°F, it would only be possible to reach a higher temperature from the heat gained in the test section and heat gains through the insulation. It was also found at a later stage that the system was not capable of inducing a step-change at the wall. The wall temperature, rather than follow a step-change, followed a power law.

3.3 EXPERIMENTS

3.3.1 Test Conducted Without Flow

In order to compare the theoretical results with experimental values a series of five tests was conducted without water flowing through the test section. First, all the water was drained from the

test section a few hours before the actual test. This was done to attain a proper starting point for the test. The water in the header tank was then cooled 12 hours in advance. At the time of the test the control mechanism was set at the desired temperature for the coolant and the pump was turned on. After waiting a few minutes to ensure stabilization of the temperature in the jacket surrounding the test section, the outlet valve on the header tank was turned wide open and closed again at the time water was seen coming out of the drain hose. As soon as the valve was closed the time was noted down. This time was considered as the time datum. For the first five minutes a reading with the ice profilometer was made every minute. From that point in time measurements were made every two minutes until the ice in the test section had reduced the diameter to the point where the profilometer could not be used effectively anymore. At this point in time the profilometer was withdrawn from the test section and the water in the test section drained along with the coolant in the jacket surrounding the test section. This allowed the ice to melt and set the stage for another experiment.

3.3.2 Test Conducted With Flow

The next four tests were executed to determine the influence of flow and superheat on the rate of ice formation. Each of the four tests was carried out with a different flow rate. The flow was determined by weighing the water that exited through the rubber hose over a known period. This weighing procedure was repeated every five minutes

in order to obtain an average: at the same time the temperature of the water was taken. The test procedure was quite similar to that of the tests without flow. The desired temperature in the test section jacket was set on the control mechanism, the pump was started and allowance was made for the system to come to equilibrium. At this point the outlet valve on the header tank was opened to the desired position and measurements of the ice thickness were begun. Since it had been found in the previous experiments that the ice growth was greatest at 30 inches from the leading edge the ice thickness measurements were all made at this point. Once again readings were taken every minute for the first five minutes and every second minute from there on to the end of the test. The end of the test depended on the flow. For the three fastest flow rates it came after 10 minutes of no change in the thickness of the ice. In the case of the slowest flow rate it was determined by the limit of the ice profilometer itself.

CHAPTER IV

DISCUSSION OF RESULTS

4.1 DISCUSSION OF PROBLEM WITHOUT FLOW

The first series of tests was conducted for a two fold reason. Firstly to determine the reproducibility of the tests and secondly for comparison with the theoretical results.

It was originally assumed that the tests were conducted with a stepchange at the wall of the test section. However, it was later found that the data obtained in tests with apparently different step-changes were identical in the sense that an equal amount of time was needed to reach a certain ice thickness: for an increase in the step-change there should have been a decrease in the time needed to reach this ice thickness. Going back to experimental data that was taken during a preliminary test, it was found that the wall temperature difference decreased according to a power-law. The data obtained from this experiment for the ice growth appeared to be otherwise identical with that of experiments #1 - #5, and therefore it was concluded that a theoretical comparison should take into account a wall temperature difference decreasing with time. A plot for a vs τ for different indices n is found in fig. 2.3. These curves are all for a Ste number equal to zero i.e. based on the zeroth approximation of the temperature solution. Since the ice thickness measurements of experiment #0 matched those of #1 - #5 it was assumed that the wall temperature behaved in approximately the same fashion for all six experiments. In order to

find the interface index, n, for the experiments a log-log plot was prepared of θ_W vs t. (see fig. 4.1). The available points were found to approximate the straight line shown. The corresponding interphase index n was found to be 0.234 and the equation of the line $\theta_W = 24t^{0.234}$. However, in the preliminary test the times were not recorded with great accuracy and the errors were of the order of ±1 minute. No experiments were carried out to determine the error in the temperature readings and their magnitude is not known. Considering the possible errors in the time estimates it would be more acceptable to state that the value of n was somewhere in the range of 0.15 - 0.25.

The experimental data from experiments #1 - #5 were plotted in a graph of a vs τ (see fig. 4.2). This graph also contains the theoretical curves for n = 0 and n = 0.20.

The experimental readings on the scale were only accurate to ±1/4 of a scale division. The divisions correspond to readings in inches (see fig. 3.3) and thus the magnitude of the possible experimental errors can be found. Considering the experimental data it can be concluded that the points of the five experiments at corresponding times were within experimental error of each other: and thus the experiments were reproducible.

Comparing the points with the theoretical curves as shown in fig. 4.2 there are three factors which are mainly responsible for deviation from the theory. The first one is the above mentioned uncertainty. The second factor, the superheat ratio $\theta_{c_W}/\theta_{c_I}$, is a measure of the superheat present in the water. The superheat ratio is a ratio of θ_{c_W} the number of degrees the water is above the freezing temperature, and

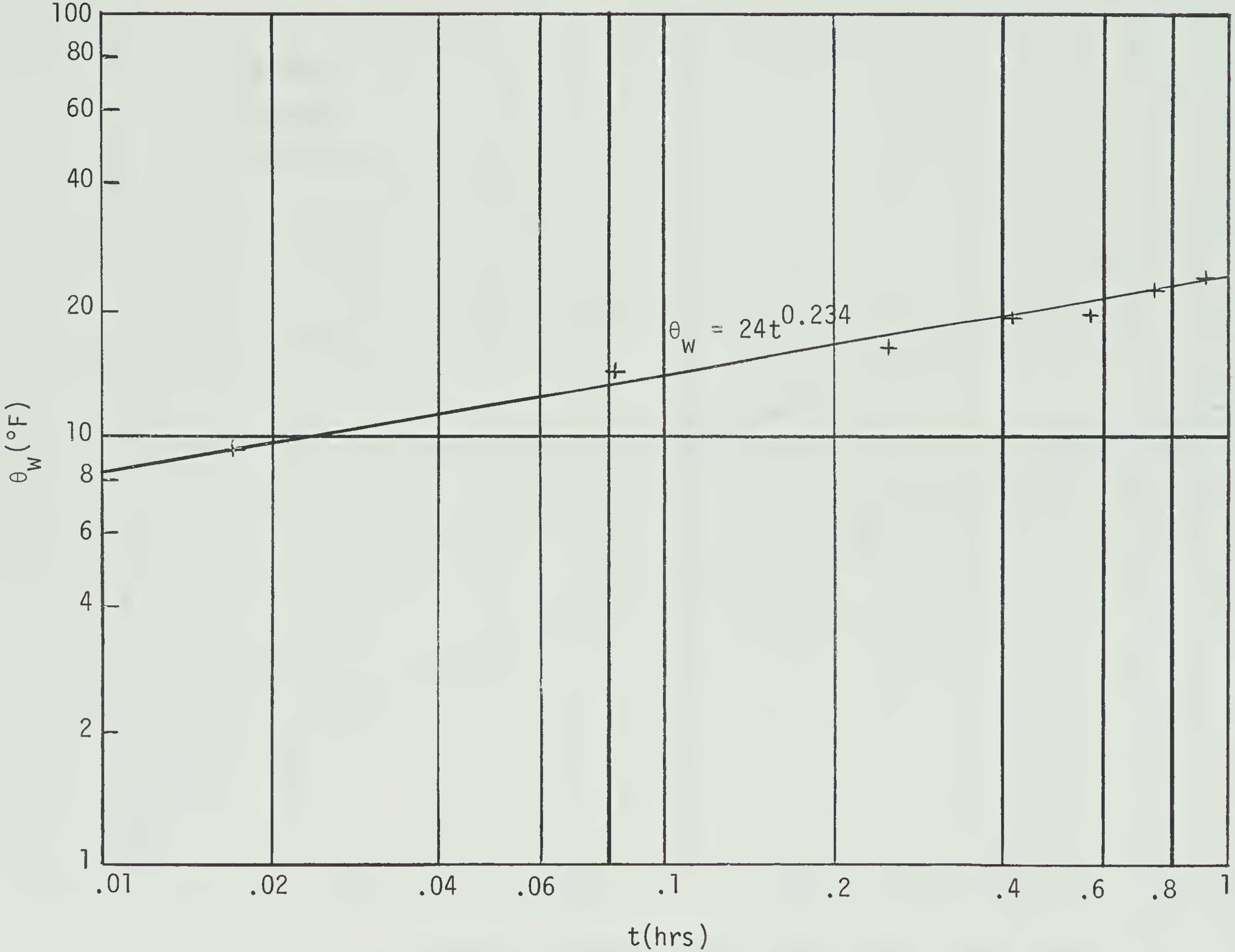

FIG. 4.1 VARIATION OF WALL TEMPERATURE WITH TIME

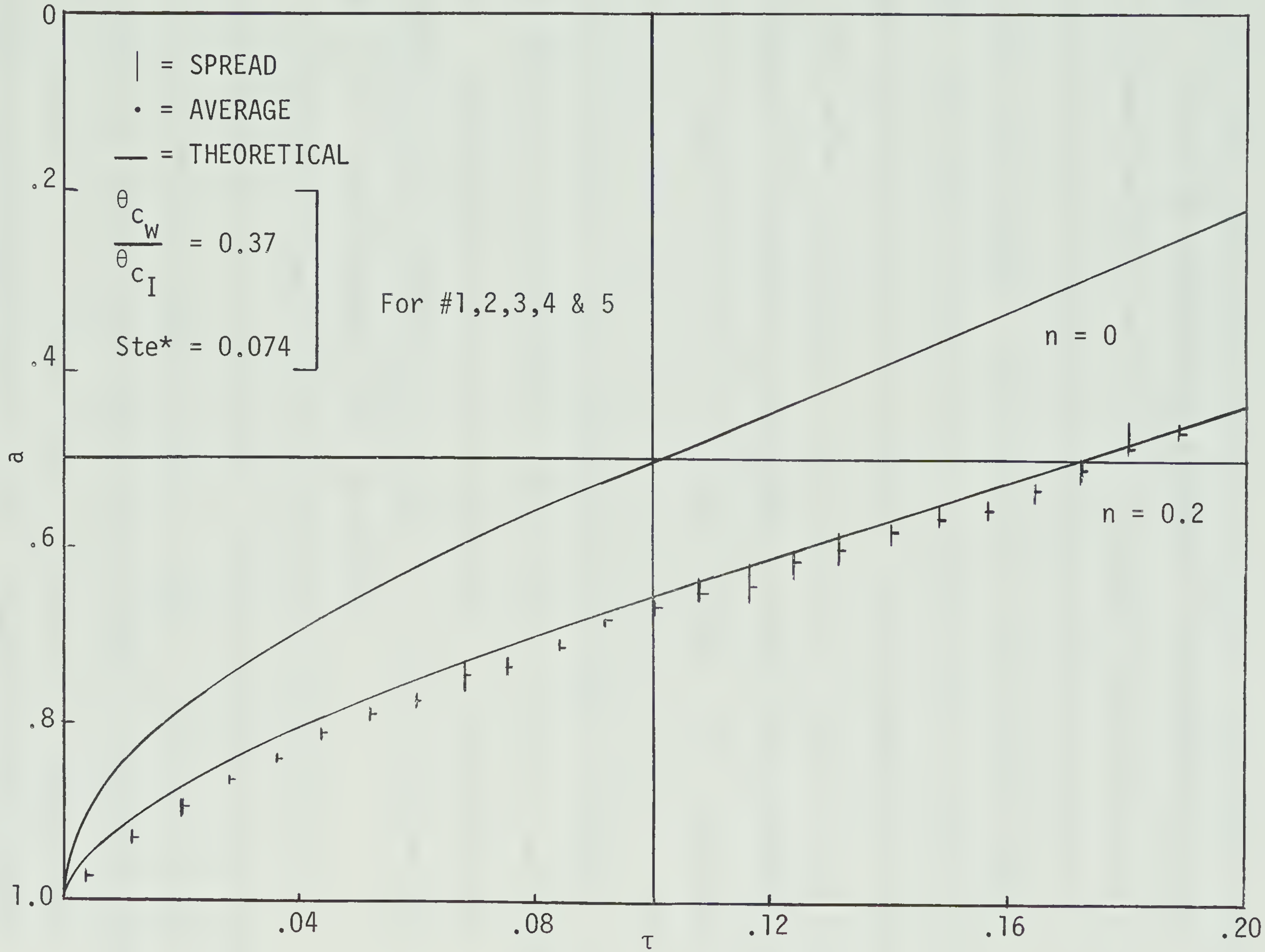

FIG. 4.2 INTERFACE PROFILES FOR EXPERIMENTS WITHOUT FLOW

θ_{c_I}, the average temperature depression of the wall below the freezing point. It is to be expected that superheat in the water will retard the ice growth and thus the experimental points should fall below the theoretical curve. The larger the superheat ratio the greater the deviation to be expected. The third factor is the non-zero Stefan number. This would also slow the ice formation since it would take into account the sensible heat of the system. Thus the experimental points would fall below the theoretical curve for Ste $\neq 0$. Lock [7] has found that the error introduced by a Ste number of 0.074 and a superheat ratio of 0.37 is approximately 7% for the plane problem. It is not unreasonable to expect that about the same error would be introduced for the cylindrical problem, at least during the early stages of formation. This error would place the experimental points below the curve as mentioned above. Considering an error of this magnitude it is found that the experimental points lie within the range of n = 0.15 - 0.25 and thus appear to be in good agreement with the theory.

The question arises if the data presented is valid for all ice formation downstream from position 15, 30 inches from the leading edge? This was to a certain extent difficult to judge. In the preliminary test axial ice profiles were taken every 10 minutes. It appeared that the ice thickness at the two positions upstream and the one position downstream from position 15 were the same. However the test section was only 3 feet long and it was not possible to determine the ice thickness very far beyond position 15. Since the ice thickness increased only very gradually in the axial direction it can not be stated with complete certainty that the ice thickness at position 15 represented

the ice thickness which one might have encountered in an infinite pipe.

A discussion of the possible deviations from the theoretical problem would not be complete without the consideration of natural convection. The water in the test section at the start of the test procedure was at 40°F; the temperature near which the density of the water is the greatest. On cooling at the walls the temperature of the water was subsequently lowered and the value of its density decreased. As a result a circulation was initiated with the water at the wall rising and in the centre descending. Thus a natural convection flow existed in the experiments without a superimposed forced flow. The effect of such a current would again be to bring the experimental points below the theoretical curve. No experiments were carried out to determine the exact magnitude of this effect.

4.2 DISCUSSION OF PROBLEM WITH FLOW

The data obtained from the experiments with flow is plotted in fig. 4.3. Four tests were undertaken, each with a different flow rate. The flows ranged from 0.4 gpm to 1.7 gpm. This data was used to determine the value of Re for each of the experiments. The number was based on the pipe diameter at the entrance of the test section and thus did not change during the experiments. In the test with the highest flow rate the Re was 1230, well below the expected value for transition to turbulence. Even at the equilibrium diameter, that is after the ice had ceased to grow in thickness at a given position, in this case position 15, the Re was only 1560 also below the expected transitional

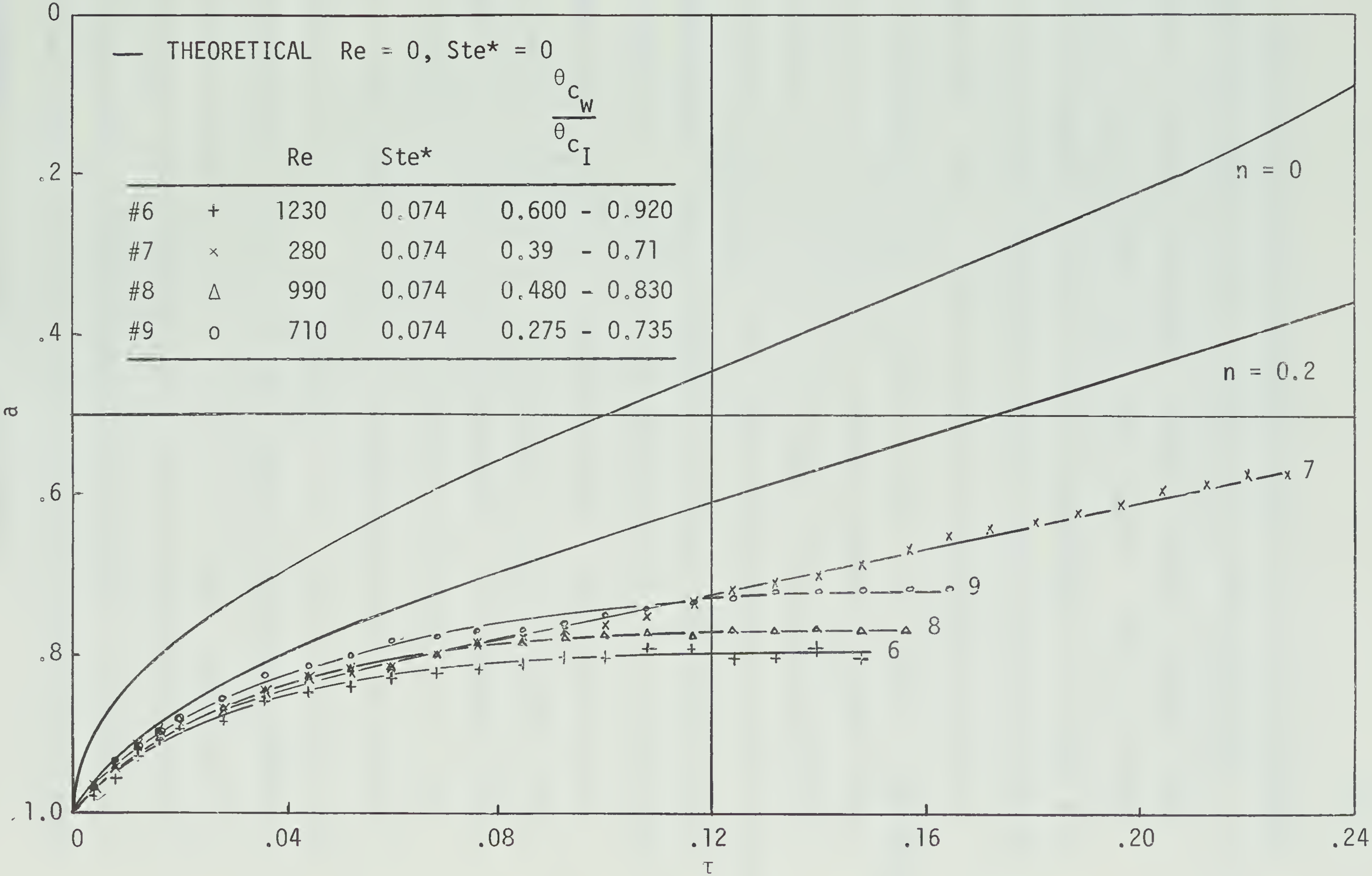

FIG. 4.3 INTERFACE PROFILES FOR EXPERIMENTS WITH FLOW

value. Thus the flow can be considered laminar in all the experiments.

The Stefan number was held the same for all four experiments. This was done to eliminate one possible variable from the experiment and to make the interpretation of the curves simpler. This was not the case for the superheat ratios of the four experiments. For tests # 6, 8 and 9 the exit water temperature gradually rose over the duration of the experiment. The θ_{c_I} value remained the same, however, and as a result the superheat ratios increased in value. The change in superheat ratio was in all likelihood due to the fact that the ice bank in the header tank was not able to cool sufficient of the make-up water and as a result the temperature of the water flowing through the test section increased slowly. The only exception to this was test #7 which had the lowest flow rate. In this test the temperature rose at first but then decreased again, almost to its previous level. Since only the exit temperature was obtained it is not possible to say if the same effect occurred at the entrance to the test section. It is therefore difficult to determine what caused the reversal in the trend of the temperature. The superheat ratios at the beginning of each test were in the range 0.27 to 0.60. The difference in the initial superheat ratio and the difference in Re are the main cause for the slower growth of ice for the various tests [#6 < #8 < #9]. The larger superheat ratio tends to retard the ice formation and Re is an indication of the velocity of the liquid flowing through the test section. At higher velocities more superheat per unit time is carried into the test section and consequently the ice growth is slowed down.

The most pronounced difference between the experiments with and

without flow is, however, the cessation of ice growth which takes place in experiments #6, #8 and #9. In each of these experiments the ice thickness remained constant for at least 10 minutes before the termination of the test. It must be assumed that this effect is due to the superheat present in the water. Ice formation ceases when the heat leaving the interface due to the temperature gradient in the ice equals the heat withdrawn from the water due to the temperature gradient in the water, i.e. no latent heat can be withdrawn and no ice formation takes place. As can be expected, with a higher flow rate more sensible heat enters the test section per unit time and, since the heat withdrawn by the cooling agent at a corresponding time in each test remains the same, the equilibrium ice thickness decreases with an increase in Re.

It may be asked why the ice ceased to grow with a wall temperature which decreased with time. It would be expected that as long as the wall temperature difference increased the ice thickness would increase as well. The asymptotic nature of the curves for the tests with flow can then only be explained by one of the following:

a) the increase in the superheat ratio balanced the increase in the wall temperature difference

b) the rate of increase in the wall temperature difference had become so small that its effect was not noticeable.

Since the superheat ratio was not constant for the duration of the experiments it was difficult to determine the influence of either the increase in wall temperature difference or the increase in superheat ratio in the region where the curves were asymptotic. Only their combined effect can be observed and discussed.

Zerkle and Sunderland [6] have studied the problem of ice formation for the steady state situation; that is for times beyond which no further ice formation takes place. They observed that for different flow rates and superheat ratios the ice thickness would be different. Their superheat ratios were quite different from those encountered in the experiments reported here and therefore it is difficult to make a comparison with their work.

In the flow tests, as in the no-flow tests, it is not possible to say with complete certainty that the ice thickness as measured at 30 inches from the leading edge represented the ice thickness for the fully developed flow region, if indeed this exists.

If the equilibrium ice thickness and the Reynolds number of experiment #6, #8 and #9 are used for a semilog plot of $(1 - a_\infty)$ vs $1 + Re$, where a_∞ is the equilibrium thickness, the three available points are found to fit a straight line going through the point $Re = 0$, $(1 - a_\infty) = 1$ (see fig. 4.4). It is reasonable to expect this point to be on the line since for no flow the ice should eventually occupy the whole cylinder. This correlation implies that for $Re = 280$ the equilibrium ice thickness a_∞ should be 0.625 which lies below the upper limit reached in experiment #7 and thus casts doubt on the shape of the curve. Questions about the shape of the curve also arise for small Re. One might expect the slope of the curve at this point to be zero from the physical nature of the problem. At least a very small flow would not likely keep open a small channel of flow. The curve beyond the transition from laminar to turbulent flow is in doubt too. Since no experiments were conducted in this region or in the region of small Re the exact shape of the curve

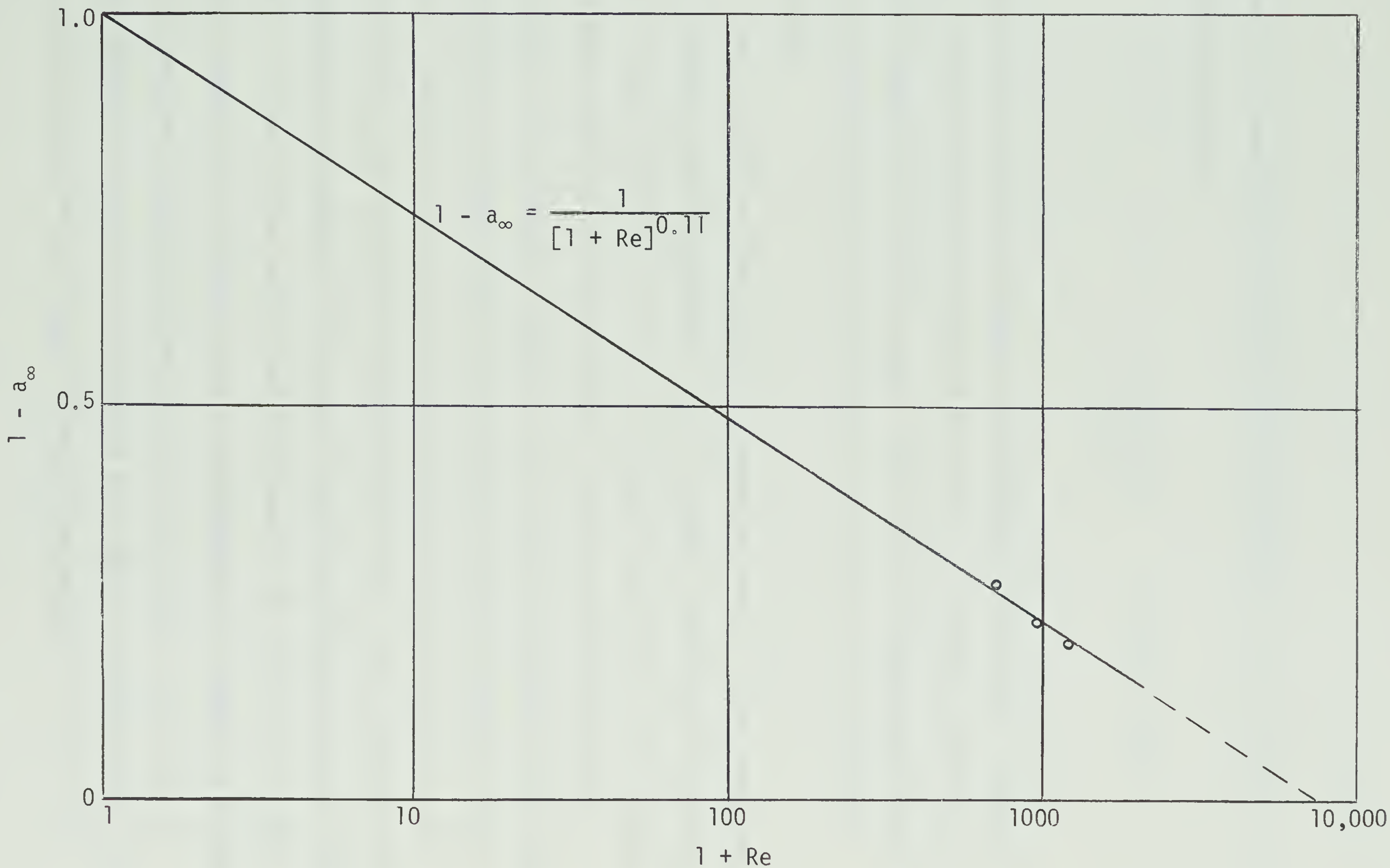

FIG. 4.4 VARIATION OF EQUILIBRIUM ICE-THICKNESS WITH REYNOLDS NUMBER

can not be determined. The equation of the straight line that fits the available points is

$$1 - a_\infty = \frac{1}{[1 + Re]^{0.11}} \qquad \text{for Ste} = 0.074$$

One may expect a family of curves for different Ste to exist. No experiments were carried out to determine the effect of natural convection on the ice formation in the flow tests. This effect is rather complicated and it is difficult to say what effect it did have on the experiments with flow.

Finally, consider the apparently anomalous test #7. The experimental data from this test as plotted in fig. 4.3 shows a deviation from the behavior of tests #6, 8 and 9. One would expect the points to lie consistently above those of the test with Re = 710. For sufficiently small times this is not the case, however, but a further examination shows that the points are situated within the uncertainty band of these two experiments. Thus the uncertainty alone may account for the deviation. This experiment was also different from the other three in the sense that the ice growth did not cease, at least until the profilometer had to be withdrawn. This may be due to the fact that because the flow rate is small the wall temperature difference may have decreased fast enough to prevent equilibrium from occurring during the span of the test. Only with a measuring device that will not interfere with the ice formation will it be possible to determine if equilibrium will be reached for this flow rate.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

A simplified theoretical approach to the problem of cylindrical heat conduction with phase change has been presented in this thesis. The agreement between theoretical and experimental results, for an ice-water system of small Stefan number and using a power-law variation of wall temperature suggests that the theory is adequate for predicting ice thickness inside a cylinder in the absence of convection.

It can also be concluded from the five experiments carried out without flow that the experiments are reproducible.

The second set of experiments was performed to observe the influence of flow and superheat on the rate of ice formation. It was found that an increase in flow rate resulted in a decrease of the equilibrium ice thickness, a_∞, in the cylinder at one particular axial station. The superheat present in the water is mainly responsible for the decrease of the equilibrium ice thickness since an increased flow rate causes an increase in the heat-input to the test section. The heat output of the test section on the other hand was for practical purposes, the same for corresponding times in the experiments regardless of the flow rate.

An empirical equation for the equilibrium ice thickness has been found to be

$$1 - a_\infty = \frac{1}{[1 + Re]^{0.11}}$$

for a Stefan number of 0.074. This equation is based on the curve drawn through the available data. It must be pointed out, however, that this curve is merely one of the many that can fit the scant data.

5.2 RECOMMENDATIONS

A great deal of work remains to be done. This series of experiments was performed keeping the Ste constant and the superheat ratio as constant as possible for all experiments. Both the effect of the Ste and of the superheat ratio on the ice formation can be studied in the future. This can be done with the equipment described earlier. However, it might be well to extend the test section in order to be certain that the ice measurements are taken in that part of the test section where the flow is fully developed.

In order to study the part of the ice formation which cannot now be measured by the ice profilometer the equipment must be redesigned. A method of measuring the ice thickness externally must be developed in order to have no obstructions to the flow and ice formation inside the test section. Only if this is done can a further evaluation of the theoretical work be made. Also the theoretical work ignores convection. With flow present this is unrealistic if the fluid temperature is not equal to the freezing temperature. The problem then must be solved taking convection into consideration.

An expanded study of experiments with a power-law variation of the wall temperature could also be undertaken for the flow and the no-flow problems to study the effect on the ice formation. More elaborate

equipment will have to be used to obtain the power-law variation of the wall temperature since the present equipment is only capable of an arbitrary variation.

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APPENDIX

FIRST ORDER ANALYSIS

From section 2.2 using equations 2.2-1 and 2.2-4 we can solve 2.2-3 as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi_1}{\partial r}\right) = \dot{c}_1 \ln r + \dot{c}_2 \qquad \text{A-1}$$

and

$$\phi_1 = \frac{1}{4}[(\dot{c}_2 - \dot{c}_1)r^2 + \dot{c}_1 r^2 \ln r] + c_3 \ln r + c_4 \qquad \text{A-2}$$

thus

$$\phi = C_1 \ln r + C_2 + \frac{Ste}{4}[(\dot{C}_2 - \dot{C}_1)r^2 + \dot{C}_1 r^2 \ln r] + \ldots \qquad \text{A-3}$$

where $C_1 = c_1 + Ste\ c_3 + (Ste)^2 c_5 + \ldots$

$C_2 = c_2 + Ste\ c_4 + (Ste)^2 c_6 + \ldots$

B.C. $r = a \qquad \phi = 0$

$r = 1 \qquad \phi = -F(\tau)$

These boundary conditions must be uniformly valid for all values of Ste. Thus take Ste = 0

$$\phi_o(a) = 0$$

$$\phi_o(1) = -F$$

and $\phi_o = c_1 \ln r + c_2$

thus

$$c_2 = -F \qquad \dot{c}_2 = -\dot{F}$$

$$c_1 = \frac{F}{\ln a} \qquad \dot{c}_1 = \frac{\dot{F}}{\ln a} - \frac{F}{a(\ln a)^2}\,\dot{a}$$

For the first approximation the boundary conditions are

$$\phi_1(a) = 0$$

$$\phi_1(1) = 0$$

Substituting in A-2 we have

$$0 = \frac{1}{4}\left[\left(-\dot{F} - \frac{\dot{F}}{\ln a} + \frac{F}{a(\ln a)^2}\,\dot{a}\right)a^2 + \left(\frac{\dot{F}}{\ln a} - \frac{F}{a(\ln a)^2}\,\dot{a}\right) \times\right.$$

$$\left. a^2 \ln a\right] + c_3 \ln a + c_4$$

and

$$0 = \frac{1}{4}\left(-\dot{F} - \frac{\dot{F}}{\ln a} + \frac{F}{a(\ln a)^2}\,\dot{a}\right) + c_4$$

therefore

$$c_4 = -\frac{1}{4}\left(-\dot{F} - \frac{\dot{F}}{\ln a} + \frac{F}{a(\ln a)^2}\,\dot{a}\right)$$

and

$$c_3 = -\frac{1}{4\ln a}\left[\dot{F} - \frac{a\dot{a}F}{\ln a} + \left(\frac{\dot{F}}{\ln a} - \frac{\dot{a}F}{a(\ln a)^2}\right)(1 - a^2)\right]$$

in general

$$\phi_m(a) = 0$$

$$\phi_m(1) = 0 \quad , \qquad \text{for } m \neq 0$$

The interface condition equation is

$$\frac{da}{d\tau} = + \left(\frac{\partial\phi}{\partial r}\right)_a$$

Substituting for ϕ using only the first two terms $\phi = \phi_o + \text{Ste}\ \phi_1$, we obtain

$$\dot{a} = \frac{F}{a \ln a} + \frac{\text{Ste}}{4}\left[\frac{a\dot{F}}{(\ln a)^2} - \frac{a\dot{F}}{(\ln a)} - \frac{\dot{F}}{a(\ln a)^2} - \frac{\dot{F}}{a \ln a}\right]$$

$$+ \frac{\text{Ste}}{4}\left[\frac{2F}{(\ln a)^2} - \frac{2F}{\ln a} + \frac{F}{a^2(\ln a)^3} - \frac{F}{(\ln a)^3}\right]\dot{a} \qquad \text{A-4}$$

using a perturbation expansion for a since Ste << 1

$$a = a_o + \text{Ste}\ a_1 + (\text{Ste})^2\ a_2 + \ldots$$

and substituting in A-4 and grouping in powers of Ste we obtain

Ste^o $$\dot{a}_o = \frac{+F}{a_o \ln a_o} \qquad \text{A-5}$$

Ste^1 $$a_o^2(\ln a_o)^3\ \dot{a}_1 + 3a_oa_1\dot{a}_o(\ln a_o)^2 + 2a_oa_1\dot{a}_o(\ln a_o)^3$$

$$= a_1(\ln a_o)^2F - 2a_1(\ln a_o)F + \frac{1}{4}\left[a_o^3(\ln a_o)\dot{F} - a_o^2(\ln a_o)^2\dot{F}\right.$$

$$\left.- a_o \ln a_o\ \dot{F} - a_o(\ln a_o)^2\dot{F}\right] + \frac{1}{4}\left[2a_o^2(\ln a_o)F\right.$$

$$\left.- 2a_o^2(\ln a_o)^2F + F - a_o^2\ F\right]\dot{a}_o \qquad \text{A-6}$$

Now

$$\frac{da_1}{d\tau} = \dot{a}_1 = \frac{da_1}{da_o}\ \frac{da_o}{d\tau} = \frac{da_1}{da_o}\ \dot{a}_o \qquad \text{A-7}$$

Substituting equations A-5 and A-7 into A-6 we obtain

$$\frac{da_1}{da_o} + \left[\frac{1 + \ln a_o}{a_o \ln a_o}\right] a_1 = \frac{1}{4}\left[\frac{a_o^2 \dot{F}}{(\ln a_o)F} - \frac{a_o \dot{F}}{F} - \frac{\dot{F}}{(\ln a_o)F}\right.$$

$$\left. - \frac{\dot{F}}{F} + \frac{2F}{(\ln a_o)^2} - \frac{2F}{\ln a_o} + \frac{F}{a_o^2(\ln a_o)^3} - \frac{F}{(\ln a_o)^3}\right] \qquad \text{A-8}$$

This is a differential equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

or a linear differential equation of the first order. The solution to this differential equation is as follows:

$$ye^{\int P(x)dx} = \int Q(x)\, e^{\int P(x)dx}\, dx + C$$

where

$$e^{\int P(x)dx} = e^{\int \frac{\ln a_o + 1}{a_o \ln a_o} da_o} = a_o \, |\ln a_o|$$

Thus

$$a_1 a_o \, |\ln a_o| = \int Q(a_o)\, a_o \, |\ln a_o| \, da_o + \text{Constant.}$$

Considerable difficulty was encountered with the integration of the above equation in particular with the term $\int \frac{a_o}{\ln a_o} \, da_o$.